

# Can One Achieve Multiuser Diversity in Uplink Multi-Cell Networks?

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## Abstract

We introduce a distributed opportunistic scheduling (DOS) strategy, based on two pre-determined thresholds, for uplink  $K$ -cell networks with time-invariant channel coefficients. Each base station (BS) opportunistically selects a mobile station (MS) who has a large signal strength of the desired channel link among a set of MSs generating a sufficiently small interference to other BSs. Then, performance on the achievable throughput scaling law is analyzed. As our main result, it is shown that the achievable sum-rate scales as  $K \log(\text{SNR} \log N)$  in a high signal-to-noise ratio (SNR) regime, if the total number of users in a cell,  $N$ , scales faster than  $\text{SNR}^{\frac{K-1}{1-\epsilon}}$  for a constant  $\epsilon \in (0, 1)$ . This result indicates that the proposed scheme achieves the multiuser diversity gain as well as the degrees-of-freedom gain even under multi-cell environments. Simulation results show that the DOS provides a better sum-rate throughput over conventional schemes.

## Index Terms

Wireless scheduling, inter-cell interference, cellular uplink, degrees-of-freedom, multi-user diversity.

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## I. INTRODUCTION

Interference between wireless links have been taken into account as a critical problem in communications. To handle intra- and inter-cell interference issues of cellular networks, a simple infinite cellular multiple-access channel model, referred to as the Wyner's model, was characterized and then its achievable throughput performance was analyzed in [1], [2]. Even if the studies in [1], [2] lead to a remarkable insight into complex and analytically intractable practical cellular environments, the model under consideration is hardly realistic. Recently, an alternative approach to showing Shannon-theoretic limits was introduced by Cadambe and Jafar [3], where interference alignment (IA) was proposed for fundamentally solving the interference problem when there are multiple communication pairs. It was shown that the IA scheme can achieve the optimal degrees-of-freedom, which are equal to  $K/2$ , in the  $K$ -user interference channel with time-varying channel coefficients. The work [3] has led to interference management schemes based on IA in various wireless network environments: multiple-input multiple-output (MIMO) interference network [4], [5], X network [6], and uplink cellular network [7]–[12].

Now we would like to consider realistic uplink networks with  $K$  cells, each of which has one base station (BS) and  $N$  mobile stations (MSs). IA for such uplink  $K$ -cell networks was first proposed in [7], but has practical challenges including a dimension expansion to achieve the optimal degrees-of-freedom.

In the literature, there are some results on the usefulness of fading in single-cell broadcast channels, where one can exploit a multiuser diversity gain: opportunistic scheduling [13], opportunistic beamforming [14], and random beamforming [15]. In single-cell downlink systems, the impact of partial channel knowledge at the transmitter has also been studied by showing whether the multiuser diversity gain can be achieved through opportunistic scheduling schemes based on limited feedback [16], [17]. Moreover, scenarios obtaining the multiuser diversity have been studied in cooperative networks by applying an opportunistic two-hop relaying protocol [18] and an opportunistic routing [19], and in cognitive radio networks with opportunistic scheduling [20]. Such opportunism can also be utilized in downlink multi-cell networks through a simple extension of [15]. In a decentralized manner, it however remains open how to design a constructive algorithm that can achieve the multiuser diversity gain in *uplink* multi-cell networks, which are fundamentally different from downlink environments since for uplink, there exists a mismatch between the amount of generating interference at each MS and the amount of interference suffered by each BS from multiple MSs, thus yielding the difficulty of user scheduling design.

In this paper, we introduce a *distributed opportunistic scheduling (DOS)* protocol, so as to show that full multiuser diversity gain can indeed be achieved in time-division duplexing (TDD) uplink  $K$ -cell networks with time-invariant channel coefficients. To our knowledge, such an attempt for the network model has never been conducted before. The channel reciprocity between up/downlink channels is utilized for every scheduling period. In the proposed scheme, based on two pre-determined thresholds, each BS opportunistically selects one MS who has a large signal strength of the desired channel link among a set of MSs generating a sufficiently small interference to other BSs, while in the conventional opportunistic algorithms [13]–[15], users with the maximum signal-to-interference-and-noise ratio (SINR) are selected for data transmission. Performance is then analyzed in terms of throughput scaling law. As our main result, it is shown that the achievable sum-rate scales as  $K \log(\text{SNR} \log N)$  in a high signal-to-noise ratio (SNR) regime, provided that  $N$  scales faster than  $\text{SNR}^{\frac{K-1}{1-\epsilon}}$  for a constant  $\epsilon \in (0, 1)$ . From the result, it is seen that the proposed scheme achieves the multiuser diversity gain as well as the degrees-of-freedom gain even under multi-cell environments. An extension to multi-carrier systems of our achievability result is also taken into account since multi-carrier modulation is an attractive choice for dynamic resource allocation as well as reduction in complexity under frequency-selective fading environments. To validate the DOS scheme for finite SNR regimes, computer simulations are performed—a better sum-rate throughput is provided over conventional schemes. Note that our protocol basically operates without global channel state information (CSI), time/frequency expansion, and iteration prior to data transmission, thereby resulting in an easier implementation. The scheme thus operates as a decentralized manner which does not involve joint processing among all communication links.

The rest of this paper is organized as follows. Section II describes the system and channel models. In Section III, the proposed DOS strategy is characterized in uplink multi-cell networks. Section IV shows its achievability in terms of sum-rate scaling. The extension to multi-carrier scenarios is described in Section V. Numerical evaluation are shown in Section VI. Finally, we summarize the paper with some concluding remark in Section VII.

Throughout this paper,  $\mathbb{C}$ ,  $\|\cdot\|$ ,  $\mathbb{E}$ , and  $\text{diag}(\cdot)$  indicate the field of complex numbers,  $L_2$ -norm of a vector, the statistical expectation, and the vector consisting of the diagonal elements of a matrix, respectively. Unless otherwise stated, all logarithms are assumed to be to the base 2.

## II. SYSTEM AND CHANNEL MODELS

Consider the interfering multiple-access channel (IMAC) model in [7], which is one of multi-cell uplink scenarios, to describe practical cellular networks. As illustrated in Fig. 1, there are multiple cells, each of which has multiple MSs. The example for  $K = 2$  and  $N = 3$  is shown in Fig. 1. Under the model, each BS is interested only in traffic demands of users in the corresponding cell. We assume that each node is equipped with a single transmit antenna and each cell is covered by one BS. The channel in a single-cell can then be regarded as the MAC. It is then possible to exploit the channel randomness and thus to obtain the opportunistic gain in multiuser environments. In this work, we do not assume the use of any sophisticated multiuser detection schemes at each receiver, thereby resulting in an easier implementation. Then, a feasible transmission scenario is that only one user in a cell transmits its data packet.<sup>1</sup>

The term  $\beta_{ik}h_{i,u_k}^{(k)} \in \mathbb{C}$  denotes the channel coefficient between user  $u_k$  in the  $k$ -th cell and BS  $i$ , consisting of the large-scale path-loss component  $0 < \beta_{ik} \leq 1$  and the small-scale complex fading component  $h_{i,u_k}^{(k)}$ , where  $u_k \in \{1, \dots, N\}$  and  $i, k \in \{1, \dots, K\}$ . For simplicity, we assume that receivers (MSs) in the same cell experience the same degree of path-loss attenuation. Especially, when  $k = i$ , the large-scale term  $\beta_{ik}$  is assumed to be 1 since it corresponds to the intra-cell received signal strength, which are much stronger than the inter-cell interference. The channel is assumed to be Rayleigh, having zero-mean and unit variance, and to be independent across different  $i$ ,  $u_k$ , and  $k$ . We assume a block-fading model, i.e., the channels are constant during one block (e.g., frame) and changes to a new independent value for every block. The received signal  $y_i \in \mathbb{C}$  at BS  $i$  is given by

$$y_i = h_{i,u_i}^{(i)}x_{u_i}^{(i)} + \sum_{k=1, k \neq i}^K \beta_{ik}h_{i,u_k}^{(k)}x_{u_k}^{(k)} + z_i, \quad (1)$$

where  $x_{u_i}^{(i)}$  is the transmit symbol of user  $u_i$  in the  $i$ -th cell. The received signal  $y_i$  at BS  $i$  is corrupted by the independent and identically distributed and circularly symmetric complex additive white Gaussian noise (AWGN)  $z_i \in \mathbb{C}$  having zero-mean and variance  $N_0$ . We assume that each user has an average transmit power constraint  $\mathbb{E} \left[ |x_{u_i}^{(i)}|^2 \right] \leq P$ .

## III. DISTRIBUTED OPPORTUNISTIC SCHEDULING

In this section, we introduce a DOS algorithm, under which one user in each cell is selected in the sense of achieving a power gain as well as generating a sufficiently small interference to other BSs. The selected MSs then transmit their data simultaneously. Assuming that the overall procedure of the proposed scheme is performed by using the channel reciprocity of TDD systems, it is possible for user  $u_i$  in the  $i$ -th cell to obtain all received channel links  $h_{k,u_i}^{(i)}$  by utilizing a pilot signaling sent from BSs, where  $u_i \in \{1, \dots, N\}$  and  $i, k \in \{1, \dots, K\}$ .

Similarly as in MIMO broadcast channels [21], an opportunistic feedback strategy is adopted in order to significantly reduce the amount of feedback overhead, which does not cause any performance loss

<sup>1</sup>Note that under the model, it is sufficient to achieve full degrees-of-freedom gain with single user transmission per cell.

compared to the full feedback scenario. The objective of our scheduling algorithm is to find a certain user out of  $N$  users in the  $i$ -th cell satisfying the following two criteria<sup>2</sup>:

$$\left| h_{i,u_i}^{(i)} \right|^2 \geq \eta_{\text{tr}} \quad (2)$$

and

$$\sum_k \beta_{ki}^2 \left| h_{k,u_i}^{(i)} \right|^2 \text{SNR} \leq \eta_I \quad (3)$$

for  $i \in \{1, \dots, K\}$  and  $k \in \{1, \dots, i-1, i+1, \dots, K\}$ , where  $\eta_{\text{tr}}$  and  $\eta_I$  denote pre-determined positive thresholds before data transmission. In particular, the value  $\eta_I > 0$  is set to a small constant independent of  $N$ , to assure the cross-channels of the target user that are in deep fade. Suitable values on  $\eta_{\text{tr}}$  and  $\eta_I$  will be specified in the later section. After computing the two metrics in (2) and (3), representing the signal strength of the desired channel link and the sum power of  $K-1$  generating interference signals to other BSs, respectively, the users such that the criteria are satisfied request transmission to their home cell BS  $i$ . Thereafter, BS  $i$  randomly selects the one among the users who send their requests to the corresponding BS, and the selected user in each cell starts to transmit its data packets. As long as the number of users per cell,  $N$ , scales faster than a certain value, there is no such event that no user in a certain cell satisfies the two criteria, which will be analyzed in Section IV.

At the receiver side, each BS detects the signal from its home cell user, while treating inter-cell interference as noise.

#### IV. ANALYSIS OF SUM-RATE SCALING LAW

In multi-cell environments, performance on the total throughput is severely limited due to the inter-cell interference especially in the high SNR regime. In this section, we show that our DOS protocol asymptotically achieves full multiuser diversity gain, i.e.,  $\log \log N$  improvement on the sum-rate performance, even at high SNRs, by deriving an achievable sum-rate scaling law. The achievability is conditioned by the scaling behavior between the number of per-cell users,  $N$ , and the received SNR. That is, we analyze how  $N$  scales with SNR so as to achieve the logarithmic gain as well as the degrees-of-freedom gain.

Let  $R_{u_i}^{(i)}(\text{SNR})$  denote the transmission rate of user  $u_i \in \{1, \dots, N\}$  in the  $i$ -th cell ( $i = 1, \dots, K$ ). Assuming inter-cell interference to be Gaussian, the rate  $R_{u_i}^{(i)}$  is then lower-bounded by

$$R_{u_i}^{(i)}(\text{SNR}) \geq \log(1 + \text{SINR}_{i,u_i}), \quad (4)$$

where  $\text{SINR}_{i,u_i}$  denotes the SINR at BS  $i$  from user  $u_i$ 's transmission and is represented by

$$\text{SINR}_{i,u_i} = \frac{\left| h_{i,u_i}^{(i)} \right|^2 P}{N_0 + \sum_k \beta_{ik}^2 \left| h_{i,u_k}^{(k)} \right|^2 P} \quad (5)$$

for  $k \in \{1, \dots, i-1, i+1, \dots, K\}$ . Now, we would like to characterize the distribution of the sum power of  $K-1$  inter-cell interference signals, which is difficult to obtain for a general class of channel models consisting of both path-loss and fading components. Instead, for analytical convenience, we use an upper bound on the amount of inter-cell interference,  $I_{i,u_i}$ , given by

$$I_{i,u_i} = \sum_k \left| h_{i,u_k}^{(k)} \right|^2 P$$

due to the fact that  $\beta_{ik} \leq 1$  for all  $k \in \{1, \dots, K\}$ .

We start from the following lemma.

<sup>2</sup>In [22], an efficient scheduling protocol based on two pre-determined thresholds has similarly been studied in single-cell broadcast channels.

*Lemma 1:* Let  $f(x)$  denote a continuous function of  $x \in [0, \infty)$ . Then,  $\lim_{x \rightarrow \infty} (1 - f(x))^x$  converges to zero if and only if  $\lim_{x \rightarrow \infty} xf(x)$  tends to infinity.

The proof of this lemma is presented in Appendix A. Since the channel coefficient is Rayleigh, the term  $h_{i,u_k}^{(k)}$  is exponentially distributed, and its cumulative distribution function (CDF) is given by

$$\Pr \left\{ \left| h_{i,u_k}^{(k)} \right|^2 \leq x \right\} = 1 - e^{-x} \quad \text{for } x \geq 0.$$

Thus, the term  $\sum_k \left| h_{i,u_k}^{(k)} \right|^2$ , corresponding to the above upper bound  $I_{i,u_i}$ , is distributed according to the chi-square distribution with  $2(K-1)$  degrees of freedom for any  $i = 1, \dots, K$  and  $u_k = 1, 2, \dots, N$ . Let  $F(x)$  denote the CDF of the chi-square distribution with  $2(K-1)$  degrees of freedom, given by

$$F(x) = \frac{\gamma(K-1, x/2)}{\Gamma(K-1)}, \quad (6)$$

where  $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$  is the Gamma function and  $\gamma(z, x) = \int_0^x t^{z-1} e^{-t} dt$  is the lower incomplete Gamma function. Then, a lower bound on  $F(x)$  is provided in the following lemma.

*Lemma 2:* For any  $0 \leq x < 2$ , the CDF  $F(x)$  in (6) is lower-bounded by

$$F(x) \geq c_1 x^{(K-1)}, \quad (7)$$

where

$$c_1 = \frac{e^{-1} 2^{-(K-1)}}{(K-1) \cdot \Gamma(K-1)}$$

and  $\Gamma(\cdot)$  is the Gamma function.

The proof of this lemma is presented in Appendix B. We are now ready to derive the achievable sum-rate scaling for uplink  $K$ -cell networks using the proposed DOS scheme.

*Theorem 1:* Suppose that  $\eta_{tr} = \epsilon \log N$  for a constant  $\epsilon \in (0, 1)$ . Then, the DOS achieves

$$\Theta(K \log \text{SNR}(\log N)) \quad (8)$$

sum-rate scaling with high probability (whp) in the high SNR regime if  $N = \omega \left( \text{SNR}^{\frac{K-1}{1-\epsilon}} \right)$ .<sup>3</sup>

*Proof:* From the fact that  $\sum_k \beta_{ki}^2 \left| h_{k,u_i}^{(i)} \right|^2 \leq \sum_k \left| h_{k,u_i}^{(i)} \right|^2$  where  $k \in \{1, \dots, i-1, i+1, \dots, K\}$ , the event that a MS in a cell satisfies the two criteria (2) and (3) occurs with probability greater than or equal to  $F(\eta_I \text{SNR}^{-1}) e^{-\eta_{tr}}$ . The probability that such an event occurs for at least one MS in a cell is then lower-bounded by

$$1 - \left( 1 - F(\eta_I \text{SNR}^{-1}) e^{-\eta_{tr}} \right)^N. \quad (9)$$

By Lemma 1, (9) converges to 1 as  $N$  tends to infinity, if and only if

$$\lim_{N \rightarrow \infty} NF(\eta_I \text{SNR}^{-1}) e^{-\eta_{tr}} \rightarrow \infty. \quad (10)$$

From Lemma 2, the term in (10) can be lower-bounded by

$$\begin{aligned} & \lim_{N \rightarrow \infty} c_1 N (\eta_I \text{SNR}^{-1})^{K-1} e^{-\eta_{tr}} \\ &= c_1 \eta_I^{K-1} \cdot \lim_{N \rightarrow \infty} \frac{N}{\text{SNR}^{K-1}} e^{-\epsilon \log N} \\ &= c_1 \eta_I^{K-1} \cdot \lim_{N \rightarrow \infty} \frac{N^{1-\epsilon}}{\text{SNR}^{K-1}}, \end{aligned}$$

<sup>3</sup>We use the following notation: i)  $f(x) = O(g(x))$  means that there exist constants  $C$  and  $c$  such that  $f(x) \leq Cg(x)$  for all  $x > c$ . ii)  $f(x) = o(g(x))$  means that  $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = 0$ . iii)  $f(x) = \Omega(g(x))$  if  $g(x) = O(f(x))$ . iv)  $f(x) = \omega(g(x))$  if  $g(x) = o(f(x))$ . v)  $f(x) = \Theta(g(x))$  if  $f(x) = O(g(x))$  and  $g(x) = O(f(x))$  [23].

which increases with  $N$  (or equivalently SNR) as  $N$  scales faster than  $\text{SNR}^{\frac{K-1}{1-\epsilon}}$ . Hence, for each cell, there exists at least one MS satisfying (2) and (3) whp. From (4) and (5), a lower bound on the achievable sum-rate is finally given by

$$\begin{aligned}
& \sum_{i=1}^K R_{u_i}^{(i)}(\text{SNR}) \\
& \geq \sum_{i=1}^K \log \left( 1 + \frac{|h_{i,u_i}^{(i)}|^2 P}{N_0 + \sum_{k \in \{1, \dots, i-1, i+1, \dots, K\}} \beta_{ik}^2 |h_{i,u_k}^{(k)}|^2 P} \right) \\
& \geq \sum_{i=1}^K \log \left( 1 + \frac{|h_{i,u_i}^{(i)}|^2 P}{N_0 + \sum_{i=1}^K \sum_{k \in \{1, \dots, i-1, i+1, \dots, K\}} \beta_{ik}^2 |h_{i,u_k}^{(k)}|^2 P} \right) \\
& \geq K \log \left( 1 + \frac{\eta_{\text{tr}} \text{SNR}}{1 + K \eta_I} \right) \\
& \geq K \log (1 + \epsilon c_2 (\log N) \text{SNR}), \tag{11}
\end{aligned}$$

which scales as  $K \log \text{SNR}(\log N)$ , under the condition  $N = \omega \left( \text{SNR}^{\frac{K-1}{1-\epsilon}} \right)$ , where  $c_2 > 0$  is a constant. This completes the proof of this theorem. ■

Note that the logarithmic term in (8) is due to the multiuser diversity gain of the DOS. From the above theorem, the following interesting observation is made according to parameters  $\epsilon$  and  $N$ .

*Remark 1:* As  $\epsilon$  increases, the achievable sum-rate in (11) gets improved due to the increased received SNR, even if scaling laws do not fundamentally change. On the other hand, the minimum number of per-cell MSs,  $N$ , required to guarantee the achievability, also needs to scale faster. Thus, a suitable selection for the threshold  $\eta_{\text{tr}} = \epsilon \log N$  should be performed according to given network environments.

In addition, it would be worthy to show our result at finite SNRs that are practical operating regimes.

*Remark 2:* As in the high SNR case, suppose that  $\eta_{\text{tr}} = \epsilon \log N$  for a constant  $\epsilon \in (0, 1)$ . In the finite SNR regime, independent of  $N$ , the DOS then achieves

$$\Theta(K \log \log N)$$

sum-rate scaling whp if  $N = \omega(1)$ . This is because  $\Theta(K \log \text{SNR}(\log N)) = \Theta(K(\log \text{SNR} + \log \log N)) = \Theta(K \log \log N)$  (the detailed step is omitted here since the proof essentially follows that of Theorem 1).

For comparison, we now show an upper limit on the sum-rate in uplink  $K$ -cell networks.

*Remark 3:* From a genie-aided removal of all the inter-cell interference, we obtain  $K$  parallel MAC systems. Under the basic assumption that only one MS per cell transmits its data, the throughput scaling for each MAC is thus upper-bounded by  $O(\log \text{SNR}(\log N))$  (see [13]). Hence, it is seen that this upper bound on the sum-rate scaling,  $K \log \text{SNR}(\log N)$ , matches our lower bound in (8) that is achieved using our distributed algorithm based on only local CSI at each node.

## V. EXTENSION TO MULTI-CARRIER SYSTEMS

The DOS algorithm can easily be implemented in multi-carrier systems. From the fact that our work is conducted under the block-fading model, a natural way is to apply the proposed scheme to orthogonal

frequency subchannels, each of which experiences relatively flat fading. We focus on a certain subchannel. Let  $\beta_{ik} \mathbf{h}_{i,u_k}^{(k)}(n) \in \mathbb{C}^{N_{\text{sub}} \times 1}$  denote the frequency response for the  $n$ -th subchannel of the uplink channel from user  $u_k$  in the  $k$ -th cell to BS  $i$ , whose elements are assumed to be the same. Here,  $N_{\text{sub}}$  indicates the number of subcarriers in one subchannel, which has no need for tending to infinity,  $u_k \in \{1, \dots, N\}$  and  $i, k \in \{1, \dots, K\}$ . Under the multi-carrier model, the DOS scheme and its performance analysis almost follow the same steps as those shown in Sections in III and IV, respectively. The users such that the two criteria, expressed as  $\|\mathbf{h}_{i,u_i}^{(i)}(n)\|^2 \geq \eta_{\text{tr}}$  and  $\sum_k \beta_{ki}^2 \|\mathbf{h}_{k,u_i}^{(i)}(n)\|^2 \text{SNR} \leq \eta_I$ , are satisfied request transmission to their home cell BS  $i$ . Thereafter, BS  $i$  randomly selects the one among the users who send their requests.

## VI. NUMERICAL EVALUATION

In this section, we perform computer simulation to validate the performance of our DOS scheme for finite parameters  $N$  and SNR in uplink multi-cell environments. Now, we slightly modify our protocol so that it is suitable for numerical evaluation. To be specific, among the MSs satisfying the criterion (3) for a given  $\eta_I$ , the one with the maximum signal strength of the desired channel link is selected for each cell. Assuming less  $\eta_I$  reduces the inter-cell interference, but corresponds to a smaller multiuser diversity gain. On the other hand, the greater  $\eta_I$  we have, the more multiuser diversity gain it may enable to capture at the cost of increased interference. It is thus not clear whether having larger  $\eta_I$  is beneficial or not in terms of sum-rate improvement. Hence, for given parameters  $K$  and  $N$ , the value  $\eta_I$  needs to be carefully chosen for better sum-rate performance. Note that the optimal  $\eta_I$  can be numerically decided prior to data transmission and the DOS scheme operates with the optimal parameter.<sup>4</sup> In our simulation, the channels in (1) are generated  $1 \times 10^5$  times for each system parameter.

First, we show numerical results by simply assuming no large-scale path-loss component, i.e.,  $\beta_{ik} = 1$  for  $i, k \in \{1, \dots, K\}$ . In Fig. 2, the average achievable rates per cell of the proposed scheme are evaluated according to received SNRs (in dB scale) and are compared with those of the following two scheduling methods: the users having the maximum SNR value and the minimum amount of generating interference are selected for data transmission (we represent them with *MaxSNR* (maximum SNR) and *MinGI* (minimum generating interference), respectively, in the figure). More specifically, the MinGI scheme operates in the sense that BS  $i \in \{1, \dots, K\}$  selects one user such that the value  $\sum_k \beta_{ki}^2 |h_{k,u_i}^{(i)}|^2 \text{SNR}$  ( $k \in \{1, \dots, i-1, i+1, \dots, K\}$ ), shown in (3), is minimized. As an example, the simulation environments are  $K = 3$  and  $N = 100$ . The optimal  $\eta_I$  is then given by 0.5. It is shown that the DOS scheme outperforms the conventional ones for all the SNR regimes. It is also examined how efficiently we decide the threshold  $\eta_I$  in terms of maximizing the sum-rate for various system parameters. The optimal value of  $\eta_I$  is summarized in Table I. Note that given parameters  $K$  and  $N$ , the optimal  $\eta_I$  is uniquely determined regardless of the received SNR.

Second, to show the outstanding performance of the DOS scheme under practical cellular environments, we run the system-level simulation for the case where large-scale path-loss and shadowing components are incorporated into our channel model. The simulation methodology of the Third-Generation Partnership Project 2 (3GPP2) [24] is employed with a slight modification to construct a multi-cell environment. A cell is formed as a hexagon whose radius is 500 m. The cell is piled up in the nearest outer 6 cells (i.e.,  $K = 7$  is assumed). The users are randomly distributed in a uniform manner. More specific system parameters are listed in Table II. Now, the average achievable rates per cell of our scheme are evaluated according to transmit powers (in dBm scale) and are then compared with those of the other two methods, MaxSNR and MinGI. In this case, as illustrated in Fig. 3, the MaxSNR scheme has much higher rates than those of MinGI since the users having the maximum SNR value are commonly located at cell center

<sup>4</sup>Even when parameters  $K$  and  $N$  are time-variant for every transmission block, the optimal  $\eta_I$  can also be updated at each BS in a decentralized manner according to the lookup table, shown in Table I, and thus our DOS scheme performs well without any performance loss.

regions because of the large-scale fading effect, thus leading to a sufficiently small amount of inter-cell interference as well. It is also shown that the proposed DOS still outperforms the conventional schemes over all the transmit powers.

## VII. CONCLUSION

The low-complexity DOS protocol was proposed in uplink  $K$ -cell networks, where the global CSI, dimension extension, parameter adjustment through iteration, and multiuser detection are not required. The achievable sum-rate scaling was then analyzed—the DOS scheme asymptotically achieves  $\Theta(K \log(\log N))$  throughput scaling as long as  $N$  scales faster than  $\text{SNR}^{\frac{K-1}{1-\epsilon}}$  for a constant  $\epsilon \in (0, 1)$ . It thus turned out that both degrees-of-freedom and multiuser diversity gains are obtained even under multi-cell environments. Simulation results showed that the proposed DOS outperforms two conventional schemes in terms of sum-rate. The optimal threshold regarding the amount of generating interference was also examined for various system parameters under no large-scale fading assumption.

## APPENDIX

### A. Proof of Lemma 1

If  $\lim_{x \rightarrow \infty} xf(x) \rightarrow \infty$ , then it follows that  $f(x) = \omega\left(\frac{1}{x}\right)$ , thus resulting in

$$\begin{aligned} \lim_{x \rightarrow \infty} (1 - f(x))^x &= o\left(\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x\right) \\ &= o(1). \end{aligned}$$

It is hence seen that  $\lim_{x \rightarrow \infty} (1 - f(x))^x$  converges to zero. If  $\lim_{x \rightarrow \infty} xf(x)$  is finite, then there exists a constant  $c_3 > 0$  such that  $xf(x) < c_3$  for any  $x \geq 0$ . We then have

$$\begin{aligned} \lim_{x \rightarrow \infty} (1 - f(x))^x &> \lim_{x \rightarrow \infty} \left(1 - \frac{c_3}{x}\right)^x \\ &= e^{-c_3} > 0, \end{aligned}$$

which complete the proof.

### B. Proof of Lemma 2

The lower incomplete Gamma function satisfies the inequality  $\gamma(z, x) \geq \frac{1}{z}x^ze^{-x}$  for  $z > 0$  and  $0 \leq x < 1$  since

$$\begin{aligned} \gamma(z, x) &= \frac{1}{z}x^ze^{-x} + \frac{1}{z}\gamma(z+1, x) \\ &= \frac{1}{z}x^ze^{-x} + \frac{1}{z(z+1)}x^{z+1}e^{-x} + \dots \\ &\geq \frac{1}{z}x^ze^{-1}. \end{aligned}$$

Applying the above bound to (6), we finally obtain (7), which completes the proof.



## REFERENCES

- [1] O. Somekh and S. Shamai (Shitz), "Shannon-theoretic approach to a Gaussian cellular multi-access channel with fading," *IEEE Trans. Inf. Theory*, vol. 46, no. 4, pp. 1401–1425, Jul. 2000.
- [2] N. Levy and S. Shamai (Shitz), "Information theoretic aspects of users' activity in a Wyner-like cellular model," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2241–2248, Jul. 2010.
- [3] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the  $K$ -user interference channel," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.
- [4] K. Gomadam, V. R. Cadambe, and S. A. Jafar, "A distributed numerical approach to interference alignment and applications to wireless interference networks," *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, Jun. 2011.
- [5] T. Gou and S. A. Jafar, "Degrees of freedom of the  $K$ -user  $M \times N$  MIMO interference channel," *IEEE Trans. Inf. Theory*, vol. 56, no. 12, pp. 6040–6057, Dec. 2010.
- [6] V. R. Cadambe and S. A. Jafar, "Degrees of freedom of wireless  $X$  networks," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Toronto, Canada, Jul. 2008, pp. 1268–1272.
- [7] C. Suh and D. Tse, "Interference alignment for cellular networks," in *Proc. 46th Annual Allerton Conf. on Commun., Control, and Computing*, Monticello, IL, Sep. 2008.
- [8] B. C. Jung and W.-Y. Shin, "Opportunistic interference alignment for interference-limited cellular TDD uplink," *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 148–150, Feb. 2011.
- [9] B. C. Jung, D. Park, and W.-Y. Shin, "A study on the optimal degrees-of-freedom of cellular networks: Opportunistic interference mitigation," in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, pp. 2067–2071, Nov. 2010.
- [10] S.-H. Hur, B. C. Jung, and B. D. Rao, "Sum Rate Enhancement by Maximizing SGINR in an Opportunistic Interference Alignment," in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, pp. 354–358, Nov. 2011.
- [11] B. C. Jung, D. Park, and W.-Y. Shin, "Opportunistic Interference Mitigation Achieves Optimal Degrees-of-Freedom in Wireless Multi-cell Uplink Networks," *IEEE Trans. Commun.*, vol. 60, no. 7, pp. XXX–XXX, Jul. 2012.
- [12] H. J. Yang, W.-Y. Shin, B. C. Jung, and A. Paulraj, "Opportunistic Interference Alignment for MIMO IMAC: Effect of User Scaling Over Degree-of-Freedom," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Cambridge, MA, Jul. 2012, pp. 2646–2650.
- [13] R. Knopp and P. Humblet, "Information capacity and power control in single cell multiuser communications," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Seattle, WA, Jun. 1995, pp. 331–335.
- [14] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inf. Theory*, vol. 48, no. 6, pp. 1277–1294, Aug. 2002.
- [15] M. Sharif and B. Hassibi, "On the capacity of MIMO broadcast channels with partial side information," *IEEE Trans. Inf. Theory*, vol. 51, no. 2, pp. 506–522, Feb. 2005.
- [16] L. Li, M. Pesavento, and A. B. Gershman, "Downlink opportunistic scheduling with low-rate channel state feedback: Error rate analysis and optimization of the feedback parameters," *IEEE Trans. Commun.*, vol. 58, no. 10, pp. 2871–2880, Oct. 2010.
- [17] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-antenna downlink channels with limited feedback and user selection," *IEEE J. Select. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sep. 2007.
- [18] S. Cui, A. M. Haimovich, O. Somekh, and H. V. Poor, "Opportunistic relaying in wireless networks," *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp. 5121–5137, Nov. 2009.
- [19] W.-Y. Shin, S.-Y. Chung, and Y. H. Lee, "Parallel opportunistic routing in wireless networks," *IEEE Trans. Inf. Theory*, under revision for possible publication, [Online]. Available: <http://arxiv.org/abs/0907.2455>.
- [20] T. W. Ban, W. Choi, B. C. Jung, and D. K. Sung, "Multi-user diversity in a spectrum sharing system," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 102–106, Jan. 2009.
- [21] T. Tang, R. W. Heath, Jr., S. Cho, and S. Yun, "Opportunistic feedback in multiuser MIMO systems with linear receivers," *IEEE Trans. Commun.*, vol. 55, no. 5, pp. 1020–1032, May 2007.
- [22] Z. Wang, M. Ji, H. R. Sadjapour, and J. J. Garcia-Luna-Aceves, "Interference management: a new paradigm for wireless cellular networks," in *Proc. IEEE Military Commun. Conf. (MILCOM)*, Boston, MA, Oct. 2009.
- [23] D. E. Knuth, "Big Omicron and big Omega and big Theta," *ACM SIGACT News*, vol. 8, pp. 18–24, Apr.-Jun. 1976.
- [24] 3GPP2/TSG-c.R1002 Third-Generation Partnership Project 2, 2003. [Online]. Available: <http://www.3gpp2.org/>.

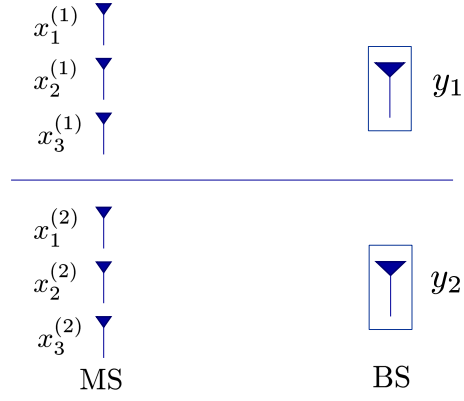


Fig. 1. The IMAC model with  $K=2$  and  $N = 3$ .

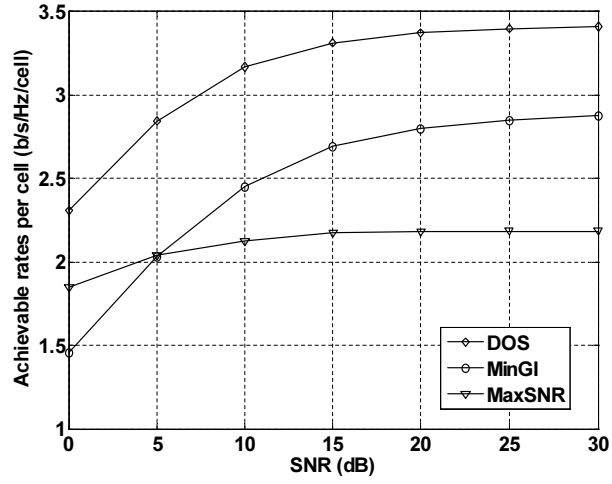


Fig. 2. The achievable rates per cell with respect to SNR, where  $\beta_{ik} = 1$  is assumed. The system with  $K = 3$  and  $N = 100$  is considered.

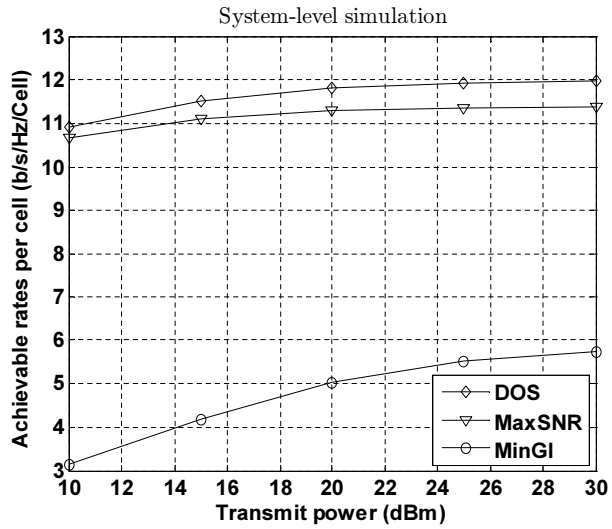


Fig. 3. The achievable rates per cell with respect to transmit power via system-level simulation. The system with  $K = 7$  and  $N = 500$  is considered.

TABLE I  
OPTIMAL VALUE OF  $\eta_I$  FOR VARIOUS SYSTEM PARAMETERS

	$N = 50$	$N = 100$	$N = 200$	$N = 300$	$N = 1000$
$K = 3$	0.7	0.5	0.4	0.3	0.2
$K = 4$	1.5	1.3	1.1	0.9	0.6
$K = 5$	2.0	1.8	1.8	1.7	1.5

TABLE II  
MULTI-CELL ENVIRONMENTS [24]

Parameter	Values
Path-loss exponent	3
Shadowing STD	8 dB
Radius of cell	500 m
Structure of cell	Hexagon
The number of MSs per cell ( $N$ )	500
User distribution	Uniform distribution on the two-dimensional network